Malaysian Journal of Mathematical Science 7(1): 17-29 (2013)

On Soft σ- Algebras

[1]Azadeh Zahedi Khameneh and [1,2]Adem Kilicman

[1] Institute for Mathematical Research,
University Putra Malaysia,
43400 UPM Serdang, Selangor, Malaysia

[2]Department of Mathematics,
Faculty of Science, Universiti Putra Malaysia,
43400 UPM Serdang, Selangor, Malaysia

E-mail: akilicman@putra.upm.edu.my

ABSTRACT

The concept of soft sets, which was first introduced by Molodstov, is a new method to deal with uncertainty. In this work, we generalize probability theory to soft probability theory. We first introduce a soft σ-algebra as a collection of soft sets over an initial universe set X with a fixed set of parameters E and show the soft σ-algebra is a parameterized family of σ-algebras over X, then present some of its properties. Finally, we discuss on the soft probability space.

Keywords: measurable spaces, probability theory, probability measure, fuzzy set, soft set, soft topology.

1. INTRODUCTION

The real world problems might be too complex to understand. Thus we need some mathematical models to simplify and understand the different aspects of real world. But most of problems in engineering, medical sciences, economics, and many more have uncertain variables and we cannot successfully use traditional tools and classical methods in these problems. To avoid difficulties in dealing with uncertainties, some theories were introduced like theory of probabilities and fuzzy theory. But they have their difficulties.

Molodtsov (1999) introduced the concept of soft sets, as a general mathematical tool for dealing with uncertain objects. A soft set gives an approximate description of an object in universe set U, and we can use any parameter including words, real number, function, and so on which we prefer.

Further, Molodtsov recommended some possible applications of the soft set theory in several directions such as smoothness of function, game theory, operations research, probability, theory of measurement, and many more.

After introduction the notion of soft set by Molodtsov (1999), this theory has been investigated increasingly from different aspects. These studies were started in 2000 by Maji *et al.* (2001, 2002, 2003, 2007), who discussed theoretical aspects and practical applications of soft sets in decision making problems, and have been continued by other authors until present time. Maji *et al.* (2001) investigated link between soft set and fuzzy set and suggested the concept of fuzzy soft set and the basic properties of it, and also (2002) applied soft sets to solve the problems in decision making. Then they (2003) introduced some new operations in soft set theory. They also (2007) applied fuzzy soft sets theory to solve decision making problems.

Aktas and Cagman (2007) introduced the basic properties of soft sets to the related concept of fuzzy sets and rough sets, and then they gave the notion of soft group and derive some basic properties of it.

Meanwhile, link between fuzzy sets, rough sets, and soft sets was investigated by some researchers (see Sun and Ma (2011), Ali (2011)). In Ali *et al.* (2009), Sezgin and Atagun (2011)), some basic theoretical properties of soft sets were studied. Similarity measure in soft set theory was studied by some authors, see Majumdar and Samanta (2008), Ali *et al.*(2009). Kharal and Ahmad (2010) introduced mapping on soft sets.

Shabir and Naz (2011) applied soft sets to propose a topological space over a universe set dealing with uncertainties. They also introduced some basic definitions and properties in topology for this new space.

Cagman *et al.* (2011) gave the concept of soft topology in different manner by Shabir and Naz (2011). They introduced the soft topology over a soft set and gave some of its related properties. More studies of topological structures associated with Shabir and Naz (2011) have been studied in (Min (2011), Aygunoglu and Aygun (2011), Hussain and Ahmad (2011), Peyghan *et al.* (2012)).

Alkhazalah *et al.* (2011) introduced two new concepts in soft set theory, soft multisets in order to investigate the problems in which we have several disjoint universe sets with different parameter sets related to each universe set, and soft expert sets Alkhazalah and Saleh (2011) to deal with decision making problems having several experts opinion of one model as soft functions and a common parameter set.

Besides working on soft sets, some authors focused on fuzzy soft set proposing first by Maji (2001) and applied this concept to introduce generalized fuzzy soft set (Tanay and Kandemir (2011)) fuzzy soft topological structures (Alkhazaleh *et al.* (2011)) and possibility fuzzy soft sets (Alkhazaleh and Saleh (2011)).

In probability theory, a probability space, which models stochastic processes, is a triplet (Ω, m, P) where Ω is a sample space, m (a collection of events) is a σ-algebra over Ω , and P is probability measure. But this theory cannot be used successfully because the events we face in our everyday experiences are related to many parameters. So they can be presented as a soft set, (Molodstov (1999)), rather than a collection of points. For example in economics cases, each event are related to some parameters, so expressing as a soft set is preferred most of the time. By using the concept of soft set, the notions of an event and its probability might be extended to soft events and its probability. But before generalized probability theory to soft probability theory, we must introduce a soft measurable space and define its properties. Therefore we need to introduce a soft σ-algebra as a collection of soft sets over an initial universe set X. In this study we introduce a soft σ-algebra, which is distinct from soft σ-algebra developed by Yinsheng (1983), and study the probability measure on it to describe the probability of occurrence soft sets.

2. PRELIMINARIES

In this section, we recall some basic definitions in soft set theory.

Definition 1. A pair (F, A) is called a soft set over U if F is a mapping given by $F : A \rightarrow P(U)$ where U refers to an initial universe, E is a set of parameters, $P(U)$ is the power set of U and $A \subseteq E$, that is, a soft set over U is a parameterized family of subsets of the universe U. (Molodtsov (1999)).

To illustrate this idea, let us consider the following example.

Example 1. Let us consider a soft set (F, E) which describes the attractiveness of houses that Mr. X is considering for purchase. Suppose that there are 4 houses in the universe $U = \{h_1, h_2, h_3, h_4\}$ under consideration, and that $E=\{e_1, e_2, e_3, e_4, e_5\}$ is a set of decision parameters. The e_i $(i = 1, \ldots, 5)$ stand for the parameters expensive, beautiful, wooden, cheap, and in green surroundings respectively. Let $F(e_1)=\{h_1, h_3\}$, $F(e_2)=\{h_1, h_3, h_4\}$, $F(e_3) = U$, $F(e_4) = \{h_2, h_4\}$, and $F(e_5) = \emptyset$. Then we can view the soft set (F, E) as consisting of the following collection of approximations: (F, E) $=\{$(expensive houses, $\{h_1, h_3\}$), (beautiful houses, $\{h_1, h_3, h_4\}$), (wooden houses, U), (cheap houses,$\{h_2, h_4\}$), (in the green surroundings, $\emptyset$)$\}$.

The following definitions were introduced by Ali *et al.* (2009).

Definition 2. For two soft sets (F, A) and (G, B) over a common universe U where $A, B \subseteq E$, (F, E) is named a soft subset of (G, E) and shown by $(F, A) \widetilde{\subseteq} (G, B)$ if:

i. $A \subseteq B$,
ii. $\forall e \in A, F(e) \subseteq G(e)$.

Definition 3. Two soft sets (F, A) , (G, B) over a common universe U are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

Definition 4. The relative complement of a soft set (F, A) is denoted by $(F, A)^r$ and is defined by $(F, A)^r = (F^r, A)$ where $F^r : A \to P(U)$ is a mapping given by $F^r(\alpha) = U - F(\alpha)$ for all $\alpha \in A$.

Definition 5. A soft set (F, A) over U is said to be a null soft set denoted by Φ, if $\forall \alpha \in A$, $F(\alpha) = \emptyset$.

But, because a null soft set is not unique and it depends on $A \subseteq E$, we prefer to use Φ_A instead of Φ for the null soft set of (F, A) and use the notation Φ to show the null soft set of (F, E).

Definition 6. A soft set (F, A) over U is said to be absolute soft set denoted by $\tilde{A}$, if $\forall \alpha \in A$, $F(\alpha) = U$.

If $A = E$, we denote the absolute soft set (F, E) over U by $\widetilde{U}$.

For further related concept such as union, intersection, and difference operations as well as De Morgan's laws in soft set theory we refer to Maji *et al.* (2003) and Ali *et al.* (2009), and soft topology we refer to Shabir and Naz (2011).

The next two definitions are useful and given by Kharal and Ahmad (2011).

Definition 7. Let (X, E) and (Y, E') be soft classes (the collection of all soft sets over X and Y respectively). Let $u : X \rightarrow Y$ and $p : E \rightarrow E'$ be mappings. Then a mapping $f : (X, E) \rightarrow (Y, E')$ is called soft map and defined as:

Let $B = p(A) \subseteq E'$ for a soft set (F, A) in (X, E), $(f(F, A), B)$ is a soft set in (Y, E') given by:

$$f(F, A)(\beta) = u(\bigcup_{\alpha \in A \cap p^{-1}(\beta)} F(\alpha))$$

for $\beta \in B \subseteq E'$.

Definition 8. Let (X, E) and (Y, E') be soft classes and $f : (X, E) \rightarrow (Y, E')$ be a soft mapping, which is defined in Definition 7. Let (G, C) be a soft set in (Y, E') and $D = p^{-1}(C)$. Then $(f^{-1}\ (G, C), D)$ is a soft set in the soft class (X, E), defined as

$$f^{-1}\ (G, C)(\alpha) = u^{-1}\ (G(p(\alpha)\ ))$$

for $\alpha \in D \subseteq E$, where $u : X \rightarrow Y$ and $p : E \rightarrow E'$ be mappings.

Similarly, the following concept of mapping, continuity and composition we refer (Aygunoglu and Aygun (2011)).

Definition 9. Let $(X, E), (Y, E')$, and (Z, E'') be soft classes and $f: (X, E) \rightarrow (Y, E')$ and g : (Y,E') → (Z,E") be two soft mappings. In addition, suppose that $u : X \rightarrow Y$, $v : Y \rightarrow Z$, $p : E \rightarrow E'$ and $q : E' \rightarrow E''$ are mappings. Then the composition of f and g shown by $((gof)(F, A), (qo\ p)(A))$ is a soft set over (Z, E'') given by

$$(gof)(F,A)(e) = v(u\left(\bigcup_{\alpha\in A\cap(qop)^{-1}(e)} F(\alpha)\right))$$

where $(F,A) \subseteq (X,E)$, and $e \in (qo\, p)(A)$.

3. SOFT σ-ALGEBRA

In this section, we present the concept of soft σ-algebra over the initial universe set X with a fixed parameter set E as below:

Definition 10. A collection m of soft sets over X is said to be a soft σ-algebra on X if m has the following properties:

i. $\tilde{X} \in m$
ii. If $(F,E) \in m$ then $(F,E)^{\mathrm{r}} \in m$
iii. It is closed under countable union.

If m is a soft σ-algebra in X, then (X,m,E) is called a soft measurable space, and the members of m are called the soft measurable sets in X.

In addition, if (X,m,E) is a soft measurable space, (Y,τ,E) is a soft topological space (see Shabir and Naz (2011)) and f is a mapping of (X,m,E) into (Y,τ,E), then f is said to be measurable if $f^{-1}(G,B)$ is a measurable set in (X,m,E) for every open set (G,B) in (Y,τ,E).

Example 2. Let $X = \{h_1,h_2,h_3\}$ be the initial universe and let $E = \{e_1,e_2,e_3,\ e_4\}$ be a set of decision parameters. Then $m = \{\ \tilde{X}\ ,\Phi, (F,E),(F,E)^r\ \}$ can be defined as a soft σ-algebra over X where:

$$(F,E)\ =\ \{(e_1,\{h_2,h_3\}),(e_2,\{h_1\}),(e_3,\{h_2,h_3\},(e_4,X)\}$$

is a soft set over X.

Example 3. Suppose $X = \{c_1, c_2, c_3\}$ is the initial universe and $E = \{e_1, e_2\}$ is the parameter set. Then $m = \{\tilde{X}$,Φ, $(F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E)\}$ can be defined as a soft σ-algebra over X where each (F_i, E) is a soft set which is described by following manner:

$$\begin{aligned}
(F_1, E) &= \{(e_1, \{c_1\}), (e_2, \emptyset)\} \\
(F_2, E) &= \{(e_1, \{c_1, c_3\}), (e_2, \{c_2\})\} \\
(F_3, E) &= \{(e_1, \{c_2, c_3\}), (e_2, X)\} \\
(F_4, E) &= \{(e_1, \{c_2\}), (e_2, \{c_1, c_3\})\} \\
(F_5, E) &= \{(e_1, \{c_1, c_2\}), (e_2, \{c_1, c_3\})\} \\
(F_6, E) &= \{(e_1, \{c_3\}), (e_2, \{c_2\})\}
\end{aligned}$$

Proposition 1. Let m be a soft σ-algebra in a set X. Then we derive the following facts:

i. $\Phi \in m$
ii. m is closed under countable intersection.

Proof.

(i) Since $\Phi = (\tilde{X})^r$, conditions (i),(ii) of Definition 10 imply that $\Phi \in m$.

(ii) Let $(F_i, E) \in m \; for \; i = 1,2, \dots$, then Definition 10 imply that

$(F_i, E)^r \in m$ for $i = 1,2, \dots$

$\overset{iii}{\Rightarrow} \widetilde{\cup_i} (F_i, E)^r \overset{ii}{\Rightarrow} (\widetilde{\cup_i} ((F_i, E)^r))^r \in m \xrightarrow{De\ Morgan\ laws} \widetilde{\cap_i} (F_i, E) \in m.$

Theorem 1. Let (X, E_1) , (Y, E_2) , (Z, E_3) be soft spaces or classes (the family of all soft sets over X, Y and Z respectively) and $f : (X, E_1) \rightarrow (Y, E_2)$, $g : (Y, E_2) \rightarrow (Z, E_3)$ be soft maps (see Definition 7). Then

$$(g \; o \; f)^{-1}(H, C) = f^{-1}(g^{-1}(\mathrm{H}, \mathrm{C}))$$

where $(H, C) \in (Z, E_3)$.

Proof.

Let $g\ o\ f\ :\ (X, E_1)\ \rightarrow\ (Z, E_3)$, and (H, C) be a soft set in (Z, E_3), then by Definitions 8 and 9:

$$(g\ of)^{-1}(H, C)(e)\ =\ (v\ o\ u)^{-1}(H(q\ o\ p)(e)$$

$$=\ (u^{-1}(v^{-1}(H[q\ (p(e))])))$$

$$= f^{-1}(v^{-1}(\mathrm{H}), q^{\ -1}(C))(\mathrm{e})$$

$$=\ f^{-1}(g^{-1}(\mathrm{H}, \mathrm{C}))$$

where $u: X\ \rightarrow\ Y,\ v: Y\ \rightarrow\ Z,\ p: E_1\ \rightarrow E_2\ ,\ q: E_2\ \rightarrow E_3$ are mappings , and $e \in\ (q\ o\ p)^{-1}\ (C)$.

Theorem 2. Let (Y, τ, E_2) and (Z, τ', E_3) be soft topological spaces and (X, m, E_1) be a soft measurable space. If $f: (X, m, E_1) \rightarrow (Y, \tau, E_2)$, $g: (Y, \tau, E_2) \rightarrow (Z, \tau', E_3)$ be continuous and measurable mappings respectively, then $g\ o\ f: (X, m, E_1)\ \rightarrow\ (Z, \tau', E_3)$ is a measurable map.

Proof.

Let $g\ o\ f: (X, m, E_1)\ \rightarrow\ (Z, \tau', E_3)$ and suppose (H, E_3) is a open soft set in (Z, τ', E_3) , (see Shabir and Naz (2011)), then by Definition 10:

$$(go\ f)^{-1}(H, E_3) =\ f^{-1}(g^{-1}(H, E_3)) \in\ m.$$

Theorem 3. Let m be a soft σ-algebra over X. If we define $m_\alpha = \{F(\alpha) | (F, E) \in\ m, \alpha \in E\}$, then m_α is a σ-algebra over X.

Proof.

(i) $\tilde{X}\ \in\ m \Rightarrow\ \in\ m_\alpha$, and $\Phi \in m\ \Rightarrow\ \emptyset \in m_\alpha$.

(ii) $Let\ F(\alpha) \in m_\alpha\ \Longrightarrow (\mathrm{F}, \mathrm{E}) \in m.$ So, $(\mathrm{F}, \mathrm{E})^{\mathrm{r}} \in m,$
Since $(F, E)^r = (F^r, E)$ and $F^r(\alpha)$=X - $F(\alpha)$, then $X - F(\alpha) \in m_\alpha$.

(iii) Suppose $F_i(\alpha) \in m_\alpha$ $where\ i \in I$, an index set, then:

$$(F_i, E) \in m \Longrightarrow \tilde{\bigcup_i}(F_i, E) \in m \Longrightarrow (\bigcup_i F_i(\alpha), E) \in m$$

$\bigcup_i F_i(\alpha) \in m_\alpha$.

Corollary 1. So corresponding to each parameter $\alpha \in E$, we can define a σ −algebra m_α on X. Hence a soft σ −algebra on X gives a parameterized family of σ −algebras on X.

Example 4. Consider Example 3. It can be seen that $m_{e_1} = \{X, \emptyset, \{c_1\}, \{c_2\}, \{c_3\}, \{c_1, c_2\}, \{c_1, c_3\}, \{c_2, c_3\}\}$ is a σ-algebra on X.

The converse of the Theorem 3 is not true in general.

Example 5. Let X = {a, b, c, d} be the universe set and $E = \{e_1, e_2\}$ be the set of parameters and m be a collection of soft sets over X such that $m = \{\Phi, \tilde{X}, (F_1, E), (F_2, E)\}$ where (F_1, E)=$\{(e_1, \{a, b\}), (e_2, X)\}$, $(F_2, E) = \{(e_1, \{c, d\}), (e_2, X)\}$. It can be seen that m_{e_1} = {X, $\emptyset$, {a, b}, {c, d}} and m_{e_2} = { $X, \emptyset$ } are σ-algebras over X ,while m is not soft σ −algebra on X.

4. SOFT PROBABILITY SPACE

Definition 12. A soft probability measure on m is a mapping $P : m \rightarrow [0, 1]$ such that

i. $P(\tilde{X}) = 1$
ii. $P\,(\bigcup_i (F_i, E)) = \sum_i P(F_i, E)\,, if\ (F_i, E) \tilde{\cap} (F_j\,, E) = \Phi.$

A soft probability space over X can be shown by a triplet (X, m, P) where m is a soft σ-algebra over X and P is the soft probability measure over m.

By adding the probability of each soft set, we can obtain a new interpretation of objects of the universe set X as below:

$$((F, E), P(F, E))$$

that shows not only a description of objects in X, but also presents the probability of such description.

Lemma 1. If (F,E) and (G,E) be two soft sets such that $(F,E) \tilde{\subseteq} (G,E)$, then we have:

i. $(G,E) = (F,E) \tilde{\cup} [(G,E) - (F,E)]$
ii. $(F,E) \tilde{\cap} [(G,E) - (F,E)] = \emptyset$

Proof.

It will be obtain from definitions in preliminary part.

Theorem 4. Let P be a soft probability measure, then it satisfies the following conditions:

i. $P(F,E)^r = 1 - P(F,E)$
ii. If $(F,E) \tilde{\subseteq} (G,E)$, then $P(F,E) \leq P(G,E)$

Proof.

(i) The definition implies:
$1 = P(\tilde{X}) = P[(F,E) \tilde{\cup} (F,E)^r] = P(F,E) + P(F,E)^r$

(ii) If $(F,E) \tilde{\subseteq} (G,E)$, by applying lemma 1(i), we get:
$P(G,E) = P(F,E) + P[(G,E) - (F,E)] \geq (F,E)$.

Example 6. Consider the example 3, that can be described by the following manner

$$
\begin{aligned}
((F_1,E),P(F_1,E)) &= (\{(e_1,\{c_1\}),(e_2,\emptyset)\},0.15)\\
((F_2,E),P(F_2,E)) &= (\{(e_1,\{c_1,c_3\}),(e_2,\{c_2\})\},0.9)\\
((F_3,E),P(F_3,E)) &= (\{(e_1,\{c_2,c_3\}),(e_2,X)\},0.85)\\
((F_4,E),P(F_4,E)) &= (\{(e_1,\{c_2\}),(e_2,\{c_1,c_3\})\},0.1)\\
((F_5,E),P(F_5,E)) &= (\{(e_1,\{c_1,c_2\}),(e_2,\{c_1,c_3\})\},0.6)\\
((F_6,E),P(F_6,E)) &= (\{(e_1,\{c_3\}),(e_2,\{c_2\})\},0.4)
\end{aligned}
$$

that give not only some descriptions of elements of X, but also indicate the possibility of such descriptions.

Example 7. Let X be an universal set and $E = \{e_1, e_2, \dots, e_n\}$ be the set of parameters. Let m be a soft σ −algebra over X. Then theorem (3) gives rise to m_{e_i} are σ-algebras over X. Take (X, m_{e_i}, P_{e_i}) as a probability space corresponding to each parameter e_i over X such that, $P_{e_i} : m_{e_i} \rightarrow [0, 1]$.

We define the soft probability measure over m by mapping: $P : m \rightarrow [0, 1]$ such that, $P(F,E) = \frac{1}{n}[\sum_{i=1}^{n} P_{e_i}(F(e_i))$, where P_{e_i} is the probability function defining over m_{e_i} and n is the number of elements in E. It is obvious that P is a soft probability measure over m.

5. CONCLUSION

In this study, we introduce the concept of "soft σ-algebra" over a collection of all soft sets of an initial universe set X with a fixed set of parameters E, and then generalized some basic properties of classic σ-algebra in to soft σ-algebra. Then we prove that each soft σ-algebra on X gives a parameterized family of σ-algebras over X. Finally, the concept of soft probability space was introduced.

REFERENCES

Aktas, H. and Cagman, N. 2007. Soft sets and soft groups. *Information Sciences*. **177**: 2726-3332.

Alkhazaleh, S., Saleh, A. R. and Hassan, N. 2011. Soft multisets theory. *Applied Mathematical Sciences*. **5**(72): 3561-3573.

Alkhazaleh, S. and Saleh, A. R. 2011. Soft expert sets. *Advances in Decision Sciences*. Article ID 757868.

Alkhazaleh, S., Saleh, A. R. and Hassan, N. 2011. Possibility fuzzy soft set. *Advances in Decision Sciences.* Article ID 479756.

Ali, M. I., Feng, F., Liu, X., Min,W. K. and Shabir, M. 2009. On some new operations in soft set theory. *Computers and Mathematics with applications*. **57**(9): 1547-1553.

Ali, M. I. 2011. A note on soft sets, rough soft sets and fuzzy soft sets. *Applied Soft computing*. **11**: 3329-3332.

Aygunoglu, A. and Aygun, H. 2011. Some notes on soft topological spaces. *Neural Comput. and Appl.* DOI 10.1007/s00521-011-0722-3.

Cagman, N., Karatas, S. and Enginoglu, S. 2011. Soft topology. *Computers and Mathematics with applications.* **62**: 351-358.

Hussain, S. and Ahmad, B. 2011. Some properties of soft topological spaces. *Computers and Mathematics with Applications.* **62**: 4058-4067.

Kharal, A. 2010. Distance and Similarity Measures for Soft sets. *New Mathematics and Natural Computation.* **6**(3): 321-334.

Kharal, A. and Ahmad, B. 2011. Mappings on soft classes. *New Mathematics and Natural Computation.* **7**(3): 471-481.

Maji, P. K., Biswas, R. and Roy, A. R. 2003. Soft Set Theory. *Computers and Mathematics with applications.* **45**: 555-562.

Maji, P. K. and Roy, A. R. 2002. An application of soft sets in a decision making problem. *Computers and Mathematics with applications.* **44**: 1077-1083.

Maji, P. K., Biswas, R. and Roy, A. R. 2001. Fuzzy Soft Set. *Journal of Fuzzy Mathematics.* **9**(3): 589-602.

Majumdar, P. and Samanta, S. K. 2008. Similarity measure of soft sets. *New Mathematics and Natural Computation.* **4**(1): 1-12.

Majumdar, P. and Samanta, S. K. 2010. Generalised fuzzy soft sets. *Computers and Mathematics with Applications.* **59**: 1425-1432.

Min, W. K. 2011. A note on soft topological spaces. *Computers and Mathematics with Applications.* **62**: 3524-3528.

Molodtsov, D. 1999. Soft set theory-first results. *Computers and Mathematics with applications.* **37**: 19-31.

Peyghan, E., Samadi, B. and Tayebi, A. 2012. On Soft Connectedness. Math GN. arXiv:1202.1668.

Roy, A. R. and Maji, P. K. 2007. A fuzzy Soft Set theoretic approach to decision making problems. *Journal of Computational and Applied Mathematics.* **203**: 412-418.

Sezgin,A. and Atagun, A. O. 2011. On operations of soft sets. *Computers and Mathematics with applications*. **61**: 1457-1467.

Shabir, M. and Naz, M. 2011. On soft topological spaces. *Computers and Mathematics with applications*. **61**: 1786-1799.

Sun, B. and Ma, W. 2011. *Soft fuzzy rough sets and its application in decision making*. Springer Science+Business Media B. V.

Tanay, B. and Kandemir, M. B. 2011. Topological structure of fuzzy soft sets. *Computers and Mathematics with Applications*. **61**: 2952-2957.

Yinsheng, Q.U. 1983. Measure of fuzzy set. *Fuzzy Sets and Systems*. **9**: 219-227.

Zadeh, L.A. 1965. Fuzzy Sets. *Information and Control*. **8**: 338-353.